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| Math 5010 |
| Zero, The Number |
| An invention of great Magnitude |
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| Zero has not always existed. This document considers the history leading up to its discovery/ invention and looks at the significance it has had, both positively and negatively, on the world. |

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Preface

Before moving on, count to ten in your head. Where was the number zero? Why didn't you start with zero? I can see why it makes sense to start with 1 when you are counting something say, sheep on the side of the road. Before there were any sheep, you really just didn't have any reason to start counting and could just say, "hey, there isn't any sheep out there"; once there was one sheep, you simply start counting with the number one. However, I asked you to just count to the number 10, there was no reference to sheep, or any object to count. If you think of the number 1 as representing a distance away from Zero, doesn’t it make sense to start by giving that reference point, the smallest distance away from zero, namely zero? I'm sure you are thinking something like, "sure, that makes sense, it's really just by convention that we start by 1, I could have just as easily started with zero."

"Easily" is definitely a relative term. It has not always been so easy to consider zero as a number. In fact, the world has resisted it, struggled with it, and fought against it; actually, quite literally. I'd have to imagine that to most people, this idea is quite out of the scope of view; which makes sense. You would have to look back a long time, and look at the overall picture of time, from before zero to now, to really understand it. Well, that is what I will be trying to do; to bring this idea into your scope of view.

So what is zero? Zero can represent the idea of nothing. In school you probably heard this a lot, “We have zero tolerance for bullying." Or, you might have heard this before, "Mom, we have no food, I'm hungry." Both of these exemplify zero being the idea of nothing. Noticing the term "zero" wasn't used in the second example, you would have to see that the word "zero" hasn't always existed, and that there were many descriptions, other words, and even feelings for it. Now compare the" idea of zero," to the "number zero." I'd hardly believe that for the trig function sine, that *sine of* (nothing) really means anything. Do you think that 15 times nothing would make sense either? To emphasize this, think back to a time before algebra, when there were just counting numbers (1, 2, 3, 4...). Tentatively think of zero as a number. Zero really seems more of an ordinal number than an actual number anyways. Wouldn't you just think of zero as being less than one?

What else could zero be? I'd guess you would have to say that zero, or better yet "0,” is a symbol. Using it, I could communicate to you the number zero, or the idea of zero. Also, consider zero as a place holder. Look at the numbers 10, 100, or 5000. Zero, is acting as a place holder. It is simply saying there are not any ones, tens, or any hundreds. So really zero is being used as part of another number to describe or symbolize it. 10, is really just one number, besides as a place holder, zero has nothing to do with that number. This may seem like an obvious idea to you, but history has shown how hard it was to connect zero, to being a place holder.

In summary: zero hasn't always existed, the world has struggled and fought against accepting it, and zero has developed through time to be an idea, a place holder, a symbol, and a number. Now that I've explained what zero is, we can now look at the actual history behind it, and go into the mathematics of it. Also, we will look and see what we have achieved because of zero.

Zero-moving through time, from nonexistence, to existence, and some of the steps along the way

Numbers have not always existed, even more so than just numbers as we know them. In a world without language at all. A long, long time ago, I'd imagine that only the most basic idea of zero existed. The cave man probably realized that they didn't have any food. Your dog even realizes that. From there, a deeper understanding of the "idea of zero" took place as the cave man surely found some way, a gesture maybe, to show that they had nothing. This is still a long ways away from the number zero, or even any other number. At this point (A point in time with no real date,) the cave men could probably only understand that they had nothing, or that they had something. I must mention here that there is simply no evidence to support anything more than this, looking this far back in time. However, observations of different cultures suggest that the cave men likely quickly developed some system to keep track of quantity. Most likely a notch on a piece of wood, or a pile of rocks. Still no actual numbers, just unit measurements of different sorts; this was the beginning of numbers. (Burton, 2006, pp.4)

As far as 30,000 B.C. there is evidence of moving past just using unit measurements. In 1937 a young wolf bone was discovered in Czechoslovakia with 55 engraved notches organized into groups of fives (Burton, 2006, pp5). Now instead of just recognizing unit measurements, groups of five were being recognized. It's worth mentioning that this grouping of the arbitrary number five, seems to stem from the fact that we have five fingers on each hand. Here it is easy to see why there would be no consideration of zero. In simple daily life, there isn't really any need for zero.

Going through the entire development of the number system would be very extensive and exhaustive. It is important to understand a brief interpretation of the development of numbers to be able to see why zero wasn't invented/discovered earlier on, and then to see why the world found it so hard, or unnecessary to accept for a very long time.

The Sumerians, made a huge step towards zero being recognized as a place holder but, I'll get to that later. For now, It is important to recognize that the Sumerians developed a decimal based numerical system, and also that they recognized the benefit of using a base 60 numerical system. They first used a reed to impress a circle or a half circle in clay to write their numbers but they eventually used a three sided stylus that could be used to make these markings: (one) or, when turned differently, (Ten.) The first used in the units place, and the second was in the 10's place. To illustrate this, observe how  represents 30. All this happened around 3000-2500 B.C., in what we now recognize as Iraq. (Kaplan, 1999, PP.7-9)

The Sumerians succumbed to the Akkadians around 2500 B.C. and then by 2000 B.C. the group became known as the Babylonians (Old Babylonians to us.) By then they still had no symbol for 5 but did have a symbol for 60 to use with their 60 base system. It is unclear how early the symbol was used, or if the Akkadians used it. The symbol was. You might be confused because isn't that the symbol for the unit/one. You are not the only ones who were confused, and this is actually what led to the Idea of a place holder, (not using the number zero as a place holder though.) Consider the number 65, which the Babylonians wrote as   . It already seems reasonably confusing to know what number is meant. Now compare the numbers 120 and 2 in Babylonian, (120),  (2). It is extremely hard to know what number is meant. Try considering the difference of 7,205 and 125. For the number 65 they might have tried leaving spaces to show the difference of the decimal places, but a space can be somewhat arbitrary and confusion still remained. Also, Spaces don't help when comparing the numbers 120 and 2. This is where the pace holder was finally recognized.

Some Babylonian between the third and sixth century B.C., constructed the sign  to be used to signify a place holder (Kaplan, 1999, pp.12). Kaplan commented on this historic innovation:

*Whoever it was, in the latter days of Babylon, that first gave to airy nothing a local habitation and a name, has left none himself. Perhaps that double wedge fittingly commemorates his place in history.* (Kaplan, 1999, pp. 13)

It was here that a place holder was discovered, but also a symbol for nothing. It is very important to recognize here though, that  does not represent a number, but nothing, or a blank. Burton seems to agree with this as he explains that this, "divider," was used from 300 B.C. on (Burton, 2006, pp.28).

Turn your attention to the Egyptian civilization which had sprouted itself along the Nile River. They are known to be one of the earliest civilizations to explore mathematics, but not really for mathematics sake, but for the geometry they needed to use. This need for geometry pushed them forward in mathematics but created a boundary in which zero was not included. In *The Biography of a Dangerous Idea,* Seife explains how the flooding Nile began their close ties with geometry and how it led them into a number sysetem. Each year the Nile flooded, the rich farmland they depended on was covered with water and dirt, destroying their property markers. After each flood, they needed a way to re-construct the property boundaries. "Rope stretchers" would go and measure out each of the boundaries. Geometry was the tool they used to accomplish this, but that tool requires numbers (Burton, 2000, pp.11).

"As early as 3500 B.C., the Egyptians had a fully developed number system that would allow counting to continue indefinitely," as long as they kept on adding new symbols anyways (Burton, 2006, pp. 14). The *Book of the Dead*, possibly from the First Dynasty, provides a recording of this number system (Burton, 2006 pp.19). In it you see that there is a unique symbol for the unit 1, for 10, 100, 1000, 10000, and so on. You might want to compare this to the timeline of the Babylonians and their innovations. The Egyptians lived approximately at the same time but they do not have any need for place holders in their numeric system. Even without place holders, they managed to develop a deep understanding of geometric math. Besides not needing place holders for this, they also never needed to consider zero for any use in geometry was not apparent to them. The Egyptians never seemed to do math, for math sake, and perhaps this is why their great mathematical enlightenment so quickly ran into walls.

Moving from around 3500B.c. to the 5th century B.C., the Greek civilization was also tied to geometry, but began looking at it for math sake. They quickly took what they learned from the Egyptians and pressed forward into the number system and mathematics. However, they clung onto their own system of numbers instead of using the Babylonian system that took advantage of decimal places and place holders that would have made dealing with fractions much easier. They considered numbers and philosophy inseparable, and looked at both of them very seriously. Perhaps too seriously when you consider what happened to Hippasus of Metapontum, which I will get to later. (Seife,2000, pp. 26)

The Greek alphabetic numeral system, developed in the 5th century B.C., was much more extensive than the Egyptians. They had different, alphabetic, symbols for the numbers 1 through 10, and then for each of the Tens up to 100 and each of the 100's up to 900. Thus they could count up to 999 with these symbols by placing them in order from highest to lowest, left to right. They could represent larger numbers though. By placing an accent mark before a number, it would indicate that that number is to be multiplied by 1000. They also introduced the myriad which could be placed next to a number to indicate that it is to be multiplied by 10000, including itself. Also, to indicate that a letter is to mean a number, they would put a bar over the letter or letters. In 150 A.D. an Alexandrian, Ptolemy, used the omicron, which is the first letter of the Greek word , meaning "nothing," to better meet the needs of doing his calculations for his work in astronomy (Burton,2006 pp.25). Many Greek astronomers by his time would use a Babylonian decimal system with place holders, but would convert their results back into their number system. However, there is still no indication that he considered it as the number zero, he just used it as a place holder.

By now we have seen how difficult and diverse the construction on numeric systems were even though there are many numeric systems that have not been mentioned. The Roman numeral system you probably are familiar with; can you think of the Roman numeral for zero? Many Cultures have come incredibly close to discovering the number zero, but none have done so yet. Even in the period from 350 to 200 B.C., with Euclid and his 5 elements, and Archimedes, the number zero was ignored and pushed aside. for the most part anyways. Interestingly enough though, there does seem to be evidence that there was consideration of the idea of the number zero, without realizing what they were doing. Zeno's paradox, around 490 B.C., and Archimedes's use of the infinite sum to calculate the area of a conic section, are excellent examples. I will go into further detail of these in the next section.

The use of a "place holder" was essentially rediscovered in India soon after Rome fell in 476 A.D. Sometime beginning in the 5th century A.D. The Indian culture moved from the Greek type numeration system to a Babylonian type of numeration system (the ideas likely spread to them as King Alexander the Great penetrated their lands in his conquest, taking parts of those cultures with him.) Note that they used the base 10 system they were familiar with, instead of adopting the base 60 system, but only used 9 numeric symbols. Zero was not used. It is confusing to know when, but at least by the 9th century A.D., the base system of 10, with a zero type "place holder" was definitely in use. It seems as though the Hindu numeral for that place holder closely resembles our number "zero." It is from these Indian/Hindu symbols that our symbols for numbers come from, which is why Seife calls for the recognition that we shouldn't be naming our numerals Arabic numerals. We should recognize them as Indian numerals. (Seife,2000, pp.66-67)

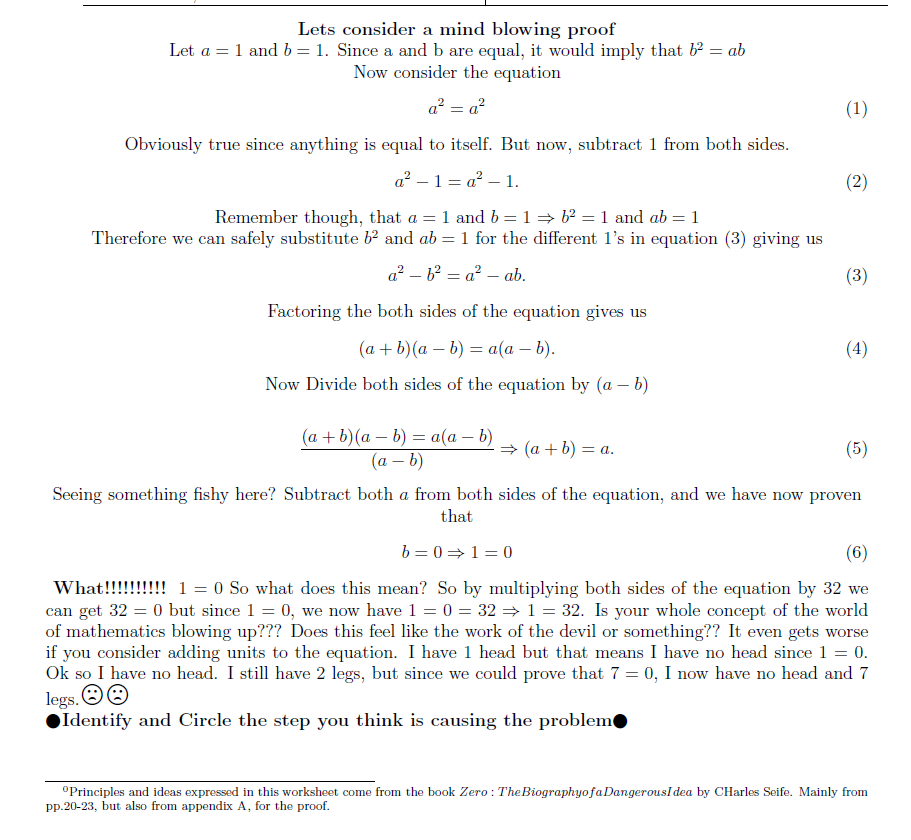
Kaplan has a few intriguing conjectures about how the symbols of place holder began to resemble what we now see as "0." The Greeks used Reckoning boards to tally sums of payment, or dept. In Apulia, the Darius Vase, dating back to the 4th century A.D., was found showing a picture of a Reckoning board; including the signs for monetary values on the board. It includes "o" to indicate one of those values. "o" is Boeotian for obal, a coin worth almost nothing. He suggests that if you didn't have a number for zero, that you might use the closest thing to it, a symbol for almost nothing. He further suggests that this symbol was rediscovered by the Indians in the dust on their counting boards. The Indians dusted their counting boards and used pebbles as counters. Perhaps the sand was used to act as a memory of what calculations had been done. The indentation of the pebble in the sand definitely resembles something familiar, and would also indicate nothing there in return. Kaplan suggests that the Greeks observed this same thing. As we know, Pythagoras and Archimedes loved to do their geometric calculations in the sand. (Kaplan, 1999, pp.23)

Going back to the Indian culture, we must recognize the importance of the lack of a tie to geometry, like the Greeks and Egyptians had. Since math didn't have to make sense geometrically something new was being considered. For instance, 5-7=-2, or better yet, 5-5=0. This seems to be the first indication that the "place holder zero" was beginning to be recognized as a number itself. It didn't take long however for the Indians to recognize the bizarre nature of zero as a number.

Considering Zero as a number, like any other number, and looking at the problems zero as a number caused, as well as the benefits that came from it

As the Indians began running into zero as a number, the bizarre properties of zero began to reveal itself. What happens when you add one number to another number? It becomes a different number, or you could say it changes and becomes larger. Add zero to any other number and it doesn't change, it stays the same. The same idea holds true with subtraction. Now, observe that by adding a number to itself enough times, it will eventually exceed any other number. This is not true with zero. It gets more bizarre when you consider the properties of multiplying numbers. All at once, multiply all the numbers on the number line by 2. What happens, the number line stretches or gets bigger. The number 1 now becomes 2. The number 5 becomes 10. But, multiply all numbers on the number line by zero at once, and the entire number line collapses to zero.

Things get far worse when you consider division. Given that a number n is multiplied by 2, we would have 2n, so how would you undo the multiplication to get the number n back? You would divide by 2, thus, . Moving forward, we know that any number, n, times Zero, equals Zero. , so then, how would you undo the multiplication to get the number 3 back? Well, there is no reason yet to say we can't divide it by zero. This would give us , but we also could say that , , and . This can't be true since all equal 0. So then this would imply that Since, we all know that a fraction just is a representation of one, specific number, this feels more than a little contradictory. What if we were to take this a little further and consider . Should this equal 1, where the zeros cancel themselves out, or should it equal zero, since anything times zero is zero? To say it at the least, accepting zero as a number, like any other number, causes you to have to deal with a lot of problems. (The Ideas and principles illustrated in this paragraph are based from Charles Seife's book in the last section of chapter 1.) Seife also gives a very pragmatic proof that shows what happens when you divide by zero just once. Below is my interpretation and depiction of his proof:



So if there were so many problems zero caused in mathematics, why was it worth it?

To explain why it is worth it, you must look at the next step in history, where algebra took form. The term algebra comes from a book called A*l-jabr,* written by al-Khowarizmi, a Muslim scholar in Bagdad. He lived from 1040 A.D to 1123 A.D., during the time of the Islamic conquest. (Seife, 2000,pp.72). The Islamic conquest covered a large part of the world, including India, where zero was already being recognized as a number. As the Islamic nation spread, they learned from the people they conquered. This is how Al-Khowarizmi learned what the Indians knew about the number zero. He took that knowledge and expanded on it to consider variables. He refined the process, developing rules and methods for solving equations, eventually writing his book, A*l-jabr*. His book has greatly impacted our numeric system, (including the numerals, notation, and rules of algebra we now use.) Fortunately enough for us, this included Zero, the number, and the symbol. (Burton, 2006, pp.250)

Imagine algebra without being able to consider any negative numbers. Not just avoiding negative solutions, but never being able to consider negative numbers at all in your work. After trying a few examples, it is quite easy to see the power zero gave Algebra as zero is like a door to the negative number. With zero comes its counterpart, infinity, which the Indians had encountered, but now with algebra, the world was really forced to face it. Struggling to know what to do with infinity eventually led to the idea of limits. The ability to understand limits was a breakthrough that lead to the foundations of a branch of mathematics called calculus, a powerful tool that lead to the innovations of the modern world.

Oddly enough that limits provide the foundation for calculus to stand on, I must first mention Calculus, (which recognized infinity,) as it was the first to be invented.

Newton-Leibniz calculus undoubtedly contributed to the creation of the modern world we are in today and deserves a great amount of recognition. But, bearing two names, who did what and who deserves more credit? A huge controversy from the beginning, still leaves people arguing today. Calculus in essence was simultaneously developed by both Newton and Leibniz, or at least there is some evidence supporting that.

Newton (1642-1727 A.D) actually discovered calculus in the undeveloped form of fluxions, in the same two years of leisure in Woolsthrope (1664-1666 A.D,)that he discovered the refractive properties of white light, and universal gravitation. The development of fluxions follows from Newton’s discovery of the binomial theorem and its use in producing an infinite series. Newton’s discovery of the binomial theorem stemmed from Wallis’s tables of calculated infinite series that today would have been written as for select integer values of n. From there, Newton considered the results of changing the upper bound to a variable. Every step along the way, newton was discovering calculus. Through mostly observation, Newton discovered the binomial theorem; although never actually proving it. (Burton,2006, pp, 391-396)

In an attempt to find a pattern of the infinite series given by Newton was able to find the area of a rectangular hyperbola of the equation to be the series expansion of the natural logarithm. Unfortunately Nicholas Mercator had already discovered the reduction of to an infinite series. In an effort to secure priority for his work, Newton showed his work to Isaac Barrow who realized the greatness of his work in its ability to solve for area of the hyperbola, but in a very general way. This in essence is the summary of calculus, the ability to solve for areas, slopes, and much more, in a general way, and Newton was realizing the Integration part of it. His work was published entitled *De Analysi per Aequationes Numero Terminorum Infinitas,* 1669 A.D.*.* In this book, without proof, he stated a rule for computing the area under the curve , which according to the rule is . To elaborate on this, Newton assumed the area z= . Using "o" as an infinitesimal, newton calculated the original line by considering an infinitesimal increase in x and in z, then using the binomial expansion, subtracting the first given equation, dividing both sides of the equation by "o," and then finally removing an terms left containing "o." This is where Newton first began to recognize the derivative process in his work. However, Newton still had no rigorous proof for the rules he used, especially for the properties of the infinitesimal, "o." (Burton,2006 pp. 417-420)

Newton first considers "o" (an infinitesimal) not as zero, in order to divide by it. Then, Newton recognizes it as zero in order to cancel out the remaining, un-needed terms left after the binomial expansion; obviously violating the laws of mathematics. Later, in an attempt to broaden his method to other curves, Newton conceived quantities differently and replaced infinitesimals with fluxions and fluents. The variable x could be a fluent, and its rate of change would be called the fluxion of the fluent, designated as . Today this could be considered as and the infinitely small increase of x over a very small period of time would be considered the moment of the fulent, denoted. This method involved the substitution of 'the fluent + the moment of the fluent,' for the fluent in an equation. However different this method appears, "o" still is assumed to be non-zero in order divide the equation, and then it is assumed to be infinitely little so that Newton could reject them. This method is much the same as with infinitesimals, but now Newton considered the change in the variables with respect to time. Still fraught with the confusion the "zero" like property was presenting, Newton attempted to remedy the illegitimate step used by changing again to using ultimate ratios. In it he attempted to explain how "o" was used as zero and not as zero by claiming that it wasn’t that the quantities “vanished,” but the rate at which they finished. Newton might have been close to recognizing limits, but was never able to explain his use of ultimate ratios in rigor. (Burton,2006 pp. 417-420)

Gottfried Leibniz (1646-1716 A.D,) about the same time as Newton, was also discovering calculus. Like Newton, the discovery seemed to stem from dabbling in infinite series. Much before his discovery, Leibniz had asserted that a “Calculus of reasoning” could be developed that would allow for the solutions of problems to be figured automatically. Without going into the debate of whether or not Leibniz came up with calculus on his own or not, between 1672 and 1676 A.D., Leibniz developed his component features and notation of his Calculus. The notation is what is most contributed to Leibniz as there is no controversy over that matter. The application of it is considered to be very thorough and user friendly. Still, in his Calculus, he advance the methods of solving for tangents. In so, he devised a method for taking the properties of tangents of a curve, and solved for the equation of the curve itself in a method of integration. His method of Integration was first developed with bars as parentheses and used the word "omn." However, "omn" was quickly ameliorated by adding the familiar ∫ to replace “omn” to mean sum. Before long, his notation shifted again to the familiar. (Burton,2006 pp. 414- 417)

Leibniz correctly determined the product rule but, like Newton, he discarded the term dxdy with the justification that it was infinitely small compared to the rest. However, it does not appear to me that in his method, the equation was ever divided by zero. Still, he showed no rigorous proof or reasoning why a term can just disappear. Using this product rule, Leibniz came up with the quotient rule and then the power rule. Even with the dispute with Newton, Newton still recognized that Leibniz method freed calculus from geometry, in addition to standardizing and generalizing the results; even though, Newton argued that the results were nothing new(Burton,2006 pp.417). It is very important to note that, like Newton, Leibniz also never recognized the concept of limits and its importance in calculus.

Both Newton’s and Leibniz methods contained results without rigorous proof, not being able to get past zero. They both came up with the same rules, and both came from a background of dealing with infinite series. The major differences are their notations and some of the steps in their methods. It’s interesting to ponder on why Newton never had the intention of publishing his discovery of calculus and whether Leibniz came about Calculus on his own accord, but we can stand on the fact that Newton discovered Calculus and Leibniz came up with a good method of notation and interpretation.

Limits, finally gave merit to calculus. It provide a reason to why Calculus works. It wasn't until Jean D'Alembert (1717- 1783 A.D.) suggests the theory of limits in a article entitled "différentiel," in his works *Encyclopédie-volume 4* in 1754, that Calculus began to have ground to stand on(Burton,2006 pp.417). However, because of his geometric reasoning, he did not have satisfactory backing of his ideas of limits. Also, this is when Zeno's paradox finally began to unravel, but I'll get into that latter. Augustine Louis Cauchy (1789-1857) finally provided a non-geometric theory of limits that appeased most of the critics of limits (Burton, 2006 pp. 604). Inspired by Lagrange's work on Taylor series expansion, Cauchy wrote the treatise *Course d' analyse de I' Ecole Royale Polytechnizue* in 1821 A.D. In it he formulates a definition of Limits. His definition was:

*When successive values attributed to a variable approach indefinitely to a fixed value so as to end by differing from it by as little as one wishes, this last is called the limit of all others.*

I would like to go into this much further, but I will leave this to you. From this you will see that Cauchy removed the geometric ties to the theory of limits, and further went on to provide the key concepts of continuity, differentiability, and the definite integral: leading much of the world to now regard him as "the creator of calculus..." (Burton, 2006, pp.606-607.)

Significance to the world, the resistance to zero

Look around you today. Consider how much religious beliefs affect everything around you. Notice the names of cities, think about the laws, the politics, and even what is considered as positive etiquette. When considering the significance of zero to the world, religion must be mention. It has had a very powerful, and very interesting role, in zero becoming a number. Right, or wrong, the Christian Church has tried and contested academic and scientific advancement. Note that the Christian church was likely not the only religious group to do this, just one worth mentioning. But, why would the church care about zero? I really can't wait to tell you, I find it quite interesting, but first, let's consider the Catholics Churches history in a little more detail.

When considering the role of the Catholic Church in Mathematics, you would probably focus on noting the negative effects the church had on Mathematics and scientific advancement. However, the church did contribute significantly in bringing mathematics to what it is today. During the darkest part of the Dark Ages("...from the Barbarian invasions of the fifth century until the eleventh century,"(Burton,2006,pp271)) where intellectual advancement was stagnant and withering away, through the Church, preservation of the ancient culture and knowledge was achieved. This was done however, unintentionally as the Church had their monasteries teach reading and writing by copying the ancient manuscripts in order to provide, simply, a literate clergy. These manuscripts contained valuable academic knowledge that would have been lost or destroyed during those Dark Ages. The churches influence can also be seen with Charlemagne. King Charlemagne (742-814 A.D.) in order to alleviate the pitiful conditions of ignorance among the clergy and civil servants, commissioned the renowned scholar Alcuin of York to be his educational advisor. Alcuin ordained that every abbey and monastery should have its own school. He instructed that they should teach the liberal arts by the quadrivium (arithmetic, geometry, astronomy, and music) and the trivium (grammar, rhetoric and logic.) He also sent out searchers after books of knowledge and learning. The king himself in Aachen attended classes of learning, along with his family. You have to remember that there was not a separation of church and state at this time. Charlemagne was only able to accomplish this with the support of the church, along with its resources. Charlemagne’s empire eventually fell but the systems of education in the monasteries remained. Never again would the possibility of literacy extinction happen again. (Burton,2006, pp. 271-272)

Fortunately the system of education was never intended for the education of a large portion of society. Without knowing it, again the church spurred on scientific advancement. The attraction of its famous cathedral schools spurred many students and teachers to seek after them, leaving an overflow of people wanting to learning and teach. Outside the walls of the cathedrals, with permission from the authorities, non-member teachers gave lectures on subjects not taught in the catholic based curriculum, including mathematics. This created a system of private teaching where education was acquired in return for paying fees. In a way the Church caused a gathering of Intellectuals where scientific/mathematical advancement could be pursued, as well as created a monetary system of education. Throughout all of this however, the church “intentionally” suppressed “new ideas,” particularly mathematics. (Burton, 2006, pp. 274)

Now for the more interesting part. The church definitely held back the advancement of Mathematics. It began with how they limited the content studied in their “schools” to the study of church doctoring and content. But from there, as mathematics began to not exactly support the church, the Church went from not supporting, to resisting. Fear of how “new ideas” might be responded to by the church has be prevalent throughout history. Since the church was not separate from the government, the power it had was immense and could reach anyone. The epitome of the defiance of the church and new ideas is with the Copernican Universe, where the sun in the center of the universe rather than the earth. Even some 300 years before the Copernican Universe (the sun at the center 1543 A.D.,) as philosophy and science were becoming preferred to the history and teachings of the church, the church began to react to the shift of educational interest. In 1210, the scientific work of Aristotle, the main cause of the shift of priorities in education, was actually forbidden to be used at the University of Paris, under pain of excommunication. (Burton, 2006, pp. 278-279)

The Church at one time even decreed it illegal to publish any book revealing “new ideas.” In response to Copernicus’s ideas and especially to the German monk Luther, and his famous list of complaints he nailed to the churches door in 1517 that began the attack on the church. The same year Copernicus died(The same year Copernicus finally dared to publish his manuscript of his book *De Revolutionibus,* (1543 A.D. ,) "Pope Paul III issued the index of forbidden books"(Seife,2000, pp.92). The church had people arrested and discredited for revealing any ideas that weren’t apparently following the churches beliefs. Giordano Bruno published *On the Infinite Universe and Worlds* in the 1580s. In it, he suggested that the earth was not the center of the universe and suggested infinite worlds. He was burned at the stake in 1600. Remember Galileo of Galilei (1564-1642 A.D.?) In 1616 the Church ordered him to cease his "scientific investigations." An unsigned notary's statement from 1616 showed an order for Galileo to not teach the Copernican views verbally or in writing (Burton, pp.344-345.) That was the same year that Copernicus's *De RevolutionibusOrbium Colestium (On the Revolution of the Heavenly spheres, 1543, according to Burton,2006 pp.344)*  was placed on the Index of forbidden books. (Seife, 2000, pp. 89-92)

So why did the church not like this scientific advancement? One reason is that they believed that God created the universe specifically for man. Thus, man is the center of the universe. Man lives on the world so without too much thought, we can see how this implies that the earth is the center of the universe. The Copernicus universe does not support this, and neither does Galileo's research using the telescope (actually not invented by him,) which supported Copernican views."Such ideas were so disturbing that there were professors at Padua who refused to credit Galileo's discoveries, refused even to look into his tellescope for fear of seeing in it things that would discredit the infallibility of Aristotle and Ptolemy, and even the Church"(Burton,2006, pp. 345). Galileo wrote a manuscript, with permission from the Church, called the *Dialogue*, and in 1632 A.D. it was allowed to be published. He was able to display the Copernican views and his evidence of it but the Church forced him to "equally" provide evidence of opposing views. Even with avoiding to declare his belief more correct than the others, when Galileo was 70 years old he had to stand trial and the *Dialogue* was placed on the Index of Prohibited Books. (Burton, 2006 pp. 344-345) Going along with the earth being the center of the universe, the church had a proof that god exists.

In addition to the belief that God was the only thing infinite, the Church actually had a proof that god existed which, the concept of zero and infinity contradicted. They believed that the earth was the center of the universe and that nothing existed outside of the universe. Since the earth, planets, and stars revolved, they understood that it took something to make them move: from their understanding of the laws of motion. They saw that the earth moved because the planets outside of it pulled on it, causing it to spin. Then, the next planet away moved because of the pull from the planets around it, and so on and so on until the outermost sphere, the sphere of stars. But what caused the outermost layer to move? There must be something making it move. The Church observed this as being a proof of god’s existence since it must be through his power that the outer most planet moves. He is the "prime mover" of the outermost layer. This proof only holds true under the Aristotelian belief that there is no such thing as the void (Zero,) and no such thing as infinity. You can see here why religion was tied so close to the number zero: zero implies infinity, infinity implies zero, but if there is infinity then there is no proof of God. (Seife,2000, pp.46-47)

To help understand why this makes sense, let Seife (pp.47) explain it in his own words, quote

*...after all, there were only two logical possibilities for the nature of the void, and both implied that the infinite exists. First, there could be an infinite amount of void-thus infinity exists. Second, there could be a finite amount of void, but since void is simply the lack of matter, there must be an infinite amount of matter to make sure that there is only a finite amount of void-thus infinity exists.*

The churches defiance of mathematics can largely be summed into a category pertaining to zero. *W*ith zero, you found infinity and nothing all at once, both questioning the existence and principles of God. Seife sums this up by saying, "As Europe slowly awakened from the Dark Ages, the void and the infinite-nothing and everything-would destroy the Aristotelian foundation of the church and open the way to the scientific revolution" (pp.83).

The Academics of science also had stakes in the matter of Zero and Infinity. One example is with the atomists. They believed that the universe is made up of Atoms. Movement of objects or people, was the movement of atoms. They theorized that for the atoms to move, there must be space for them to move, called the vacuum (the infinite Void). Without the vacuum, the atoms would just be all stuck together. Since this line of though pushed for an infinite amount vacuum, and suggested an infinite amount of matter, this opposed the Church: God created the Universe, and there is nothing outside of the universe. (Seife,2000, pp. 45)

Parmenides, a member of the Eleatic school of thought, held the belief that, "the underlying nature of the universe was changeless and immobile"(Seife,2000, pp.45). This could be viewed as supporting the churches belief, and objecting the atomists’ theory by reasoning that they have no basis, since nothing actually does move. This leads us to Zeno's paradox (450 B.C.). The Paradox goes something like this:

Achilles runs at 1 foot per second. A tortoise runs at half that speed and gets a head start of 1 foot. Let the race start. Achilles, in order to pass the tortoise, must at some point pass the one foot mark where the tortoise started, but by the time he has gotten there(1 second,) the tortoise has moved forward(half a foot.) Now he must get to where the tortoise is (the 1.5 ft. mark.) Again, it takes him time to get there, giving the tortoise time to move forward. This will continue on time after time, appearing to never end. Since he will have to do this infinitely many times, and each time takes an amount of time, how would Achilles ever catch up to the tortoise? This paradox lay unsolved for almost 2,000 years. (Seife,2000, pp. 40-43)

You can see how this supports the philosophy that nothing in the universe actually moves. The Atomists actually countered this by explaining that the atom was the smallest element and they were indivisible. Thus, there is a point when all things cannot be divided, including the space left for Achilles to catch up to the turtle. At this final step, Achilles simply hurtles an atom where the tortoise doesn't have enough time to hurtle the next one, putting the two racers even with each other. However the most provocative part of Zeno's paradox is that if you choose to accept 0/infinity, nothing in the world can move. A pretty persuasive argument as to why zero isn't actually a number and really doesn't exist at all.

I cannot finish off this section without talking about the significance zero has had with our calendar system. Have you ever noticed that looking back to 1 A.D., it follows directly after 1 B.C.. Where exactly is year zero? If you look at many publicized timelines, they even choose to ignore this fact and actually put in a year zero even though it doesn't exist. Now think about this, if someone was born on the first second of 1 A.D., they wouldn't be a 1 year old until 2 A.D.. They wouldn't be five years old until 6 A.D. and so on until when they are 100 years old. The year would then be 101 A.D. You know how keeping track of what century it is kind of confusing, is it starting to make more sense? One century is 100 years, and 100 years haven't been completed until 101 A.D., so in 101 A.D. the second century begins. In 1901, 19 centuries have been completed and the 20th century begins. A funny realization you might be making is that the 20th century ends on December 31, 2000, and the new millennium/the-21st-century begins on January 1, 2001. What day did you celebrate the new millennium? (Seife,2000, pp 56-57)

With zero being called by some as the greatest invention of all time, what exactly has zero helped us achieve?

*In the history of culture the discovery of zero will always stand out as one of the greatest single achievements of the human race.*

*-Tobias Danzig, Number:*

*The Language of Science*

*(Seife,2000, pp.12)*

It is hard to imagine the world if "zero" had never existed. I'm not really sure if I can picture what it would be like without the "idea of zero," but it helps me to see how useful it is and how prevalent. The symbol itself can be seen as a very powerful and encompassing symbol that has a lot of uses. I can picture what life would be like without the "place holder zero." School is hard enough, I would hate to have to memorize endless amounts of symbols and then have to do calculations with them. It would be impossible. Especially thing how large of numbers we are dealing with today and how frequently, especially in with computers. Large numbers were used before in Astronomy though and I have empathy for those who did their calculations without zero. I mentioned computers, but what about the binary system they operate on. I'll have to admit that I don't know if they could operate on 1s and 2s instead, but I would think that one of the numbers would still have to signify nothing(so it would really be zero.) Zero, also plays a huge role in statistics. I'm not sure if statistics would be possible without zero. That gets into zero as a number, which we have already explained how important it is in algebra and calculus, which have really led us into the modern world. I'll some it up with I'm sure is a small subset of examples of what zero has led to.

{Computers, internet, tv, "we have zero tolerance...," the number 1,0000,000,000,000,023, mathematical addition and subtraction such as , "I am hungry," the distance of 10ft makes sense, rulers, architecture, planes, anything where area or volume has to be calculated under the curve, We can move, we can win a race, realistic art(perspective goes off to infinity)}.

Appendix A

|  |  |  |
| --- | --- | --- |
| Event | Start | end |
| Notched wolf bone in groups of 5 | 30000**B.C.** |  |
| Egyptian had a fully developed number system that could be used to continue counting indefinitely | 3500 **B.C.** |  |
| Sumerians Succumbed to the Akkadians | 2500 **B.C.** |  |
| Beginning of Babylonian times | 2000 **B.C.** |  |
| Babylonian-Symbol for a Place holder-represents "nothing" | 600 **B.C.** | 200 **B.C.** |
| Greeks had an extensive alphabetic numeral system | 500 **B.C.** |  |
| Zeno was born/ little actually known about him | 450 **B.C.** |  |
| Zeno's Paradox lay unsolved, Very rough estimation | 450 **B.C.** | 1754**A.D.** |
| Ptolemy used the omicron for doing calculations of astronomy | 150 **A.D.** |  |
| Darius Vase | 300 **A.D.** | 400 **A.D.** |
| Indian culture adopted a Babylonian type numeration system | 400 **A.D.** | 500 **A.D.** |
| Dark ages | 400 **A.D.** | 1100 **A.D.** |
| Rome fell\ Place holder "re-discovered" | 476 **A.D.** |  |
| King Charlemagne | 742 **A.D.** | 814 **A.D.** |
| Indian culture was using a base 10 system with a zero type "place holder" | 800 **A.D.** | 900 **A.D.** |
| Al-Khowarizmi | 1040 **A.D.** | 1123 **A.D.** |
| Scientific work of Aristotle forbidden at the University of Paris under pain of excommunication | 1210 **A.D.** |  |
| Nicolas Copernicus | 1473 **A.D.** | 1543 **A.D.** |
| Germon monk Luther Nails list of complaints to the Church's door | 1517 **A.D.** |  |
| Copernican Universe | 1543 **A.D.** |  |
| Copernicus died/ 'De Revolutionibus Orbium colestium' was published | 1543 **A.D.** |  |
| Pope Paul II issud the index of forbidden books | 1543 **A.D.** |  |
| Galileo of Galilei | 1564 **A.D.** | 1642 **A.D.** |
| Appr. Date-Giordano Bruno published '*On the Infinite Universe and Worlds'* | 1580 **A.D.** |  |
| Giordano Bruno was burned at the stake | 1600 **A.D.** |  |
| Church ordered Galileo to cease his "scientific investigations" | 1616 **A.D.** |  |
| Copernicus's '*De RevolutionibusOrbium Colestium' was placed on the Index of forbidden books* | 1616 **A.D.** |  |
| Galileo's manuscript 'the Dalogue' | 1632 **A.D.** |  |
| Galileo stood trial for 'the Dialogue'/it was placed on the Index of forbidden books | 1635 **A.D.** |  |
| Isaac Newton | 1642 **A.D.** | 1727 **A.D.** |
| Galileo's assistant, Evangelista Torricelli, prooved that a vacume in space exists(Seife,2000, pp. 97-98) | 1643 **A.D.** |  |
| Gottfried Leibniz | 1646 **A.D.** | 1716 **A.D.** |
| Newton's years of leasure in Woolsthrope | 1664 **A.D.** | 1666 **A.D.** |
| Newton's De Analysi per Aequationes Numero Terminorum Infinitas | 1669 **A.D.** |  |
| Leibniz developed his Calculus | 1672 **A.D.** | 1676 **A.D.** |
| Jean D'Alembert | 1717 **A.D.** | 1783 **A.D.** |
| D'Alembert's Encyclopédie-volume 4 suggests the idea of limits | 1754 **A.D.** |  |
| Augusstin Louis Cauchy | 1789 **A.D.** | 1857 **A.D.** |
| Cauchy's treatise '*Course d' analyse de I' Ecole Royale Polytechnizue'* | 1821 **A.D.** |  |
| Midnight, December 31, 1999, we celebrated the New millennium on the wrong day | 1999 **A.D.** | 2000 **A.D.** |

Appendix B

**(1)** The theme of this webpage is the concept of zero. I chose this topic after being introduced to the idea that zero could be considered one of the greatest inventions of mankind and at the same time was not very easily accepted, by a trade book by Charles Seife entitled *Zero:The Biography of a Dangerous Idea.* I had previously only read a chapter or two from the book, but it really captured my attention and this project presented me with an opportunity to learn more.

**(2)** I have constructed this website in order to broaden the user's scope of view so that they will be able to see zero transcend through time. I would like users to understand what zero is, how it behaves, and make historical connections in mathematics, and the world. Zero provides a seat that can take you through almost every aspect of modern mathematics, and part of this website is designed to help make a few of those connections. More specific objectives include:

* Users observe zero in the scope of time from nonexistence to now.
* Users comprehend some of the basic properties of zero and discover how they lead to unacceptable mathematical consequences.
* Users discover the connection between zero and its religious consequences.
* Users discover proof that zero was avoided, discouraged, and fought against through time.
* Users comprehend and distinguishes between zero: an Idea of nothing, a symbol for nothing, a place holder, and a number.
* Users understand the connection zero has to infinity, calculus, and limits.
* Users visualize the historical atmosphere surrounding each step of the development of zero.

**(3)** One of the most important technological enhancements that will help achieve my objectives is the image mapping of the timeline, where the events on the timeline are linked to the material in the other web pages. Being able to use image mapping has allowed me to connect the material throughout the website in an accessible way; it has essentially tied everything together. Particularly, the timeline broadens the users point of view to see zero throughout time. Considering how much information, including background information, that is included in the webpage, the timeline will help users move through the website and focus on points that interest them. Much of the background information will be navigated around, unless the user decides to read it. This is a major component that I used to "condense" the material.

The audio is a technological enhancement that will help users be relaxed and entertained while navigating through the Explanation of Mathematics page. Receiving some feedback on the webpage, I was lead to understand that this particular page did not have very much of a natural appeal that would drive readers to read it. The audio recording allows me to add drama, emphasis, and enthusiasm for the subtle but very important aspects of zero on this page. I will also be able to clarify some of the subtle connections and more complex connections that aren't as easy to make in writing. A huge benefit of using the audio will be that users be able to consider a condense version of the written material. This will help the concept of zero remain in full view where else it could be lost in some of the more detailed descriptions.

One of my resources is a video I made of me going over Seife's proof which shows what happens when you divide by zero. I included the video so that I could take the users step by step through the proof and really emphasize the main points. This video provides a easy and interesting way for them to consider the proof as opposed to working through it on their own. The material covered in this video is a huge part of convincing the reader how scary the properties of zero can be and the consequences that happen when you consider as a number.

I chose to incorporate a technology enhanced activity about Zeno's paradox. The realm of a limit is a little out of the scope of the intent of this website but I wanted to do something to help users understand a bit of what it is and how zero is connected to it. Where this concept can be very difficult to grasp, by including the applet, I was able to provide an interactive and entertaining approach to accomplish this objective.

I incorporated a link to <http://nrich.maths.org/2671> which is about the legend of Hippasus of Metapontum, in order to provide some historical background to the severity of the atmosphere surrounding numbers. This legend is very interesting but to go into it too far would distract from the main point of the paper, and well, I’m already pushing that boundary quite a bit. I also included a variety of links in the applications page to provide visuals of the applications of zero that we used today. I am using these links to finish up the goal of showing the entire life of zero, by ending on Zero today.

**(4)** I have learned a lot through this project. I learned about the different components of zero (an Idea of nothing, a place holder, a symbol, and a number), and how they developed in a chain of events that lead to zero as we know it today. To me, one of the more interesting parts of what I learned was how the world resisted zero. It was compelling to learn how zero was tied to religion, which accounted for much of the resistance. I had no idea that zero caused so many problems for so many famous historical mathematicians, and their discomfort of zero (along with their religious ties) even prevented them from making huge strides in mathematics. It was also very interesting to see the properties of zero in a different light. I was able to learn how it would feel to be in the shoes of someone trying to accept zero when its properties defy everything I would know about numbers. I learned how big of a role zero plays in, pretty much, every aspect of mathematics. It was fun to see the artifacts left form mankind's avoidance of zero, including issues in the calendar.

For my presentation, I would like to first introduce my classmates to zero as an Idea of nothing, a place holder, a symbol, and a number. Following that I would like to have them consider some of the evidence suggesting the avoidance of zero and the repercussions of accepting zero as a number. I also want my class to observe how zero is tied to pretty much everything in mathematics. I want my classmates to see where these events are happening in time and consider how long it took for some of the steps in the development of zero to happen.

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